

Winter 2020

Updated Jan 8, 2019

AERO 550 (EECS 560) (ME 564) (CEE 571) Linear System Theory
[Scrugss] - MWF 9:30-10:30

AERO 551 (EECS 562) Nonlinear Systems and Control [Meerkov] - MW
1:30-3:00

AERO 575 Flight and Trajectory Optimization [Kolmanovsky] - MW
4:30-6:00

AERO 740 Multi-Agent Control [Panagou] -TTh 10:30-12:00

EECS 419 Electric Machines and Drives [Hofmann] - TTh 3:00-4:30

EECS 460 Control Systems Analysis and Design [Mathieu] - MW 10:30-
12:00

EECS 461 Embedded Control [Cook] - TTh 9:00-10:30

EECS 464 Hands-on Robotics [Revzen] - MW 12:00-12:30

EECS 467 Autonomous Robotics [Jenkins] - MW 9:00-10:30

EECS 535 Power Systems Dynamics and Control [Hiskens] - TTh 9:00-
10:30

EECS 560 (AERO 550) (ME 564) (CEE 571) Linear System Theory
[Scrugss] - MWF 9:30-10:30

EECS 561 (ME 561) Design of Digital Control systems [Vasudevan] -
TTh 3:00-4:30

EECS 562 (AERO 551) Nonlinear Systems and Control [Meerkov] - TTh
1:30-3:00

EECS 565 (AERO 550) Linear Feedback Control [Freudenberg] - TTh
10:30-12:00

EECS 569 Production Systems Engineering [Meerkov] - TTh 4:30-6:00

EECS 598-002 Reinforcement Learning Theory [Ying] - TTh 10:30-
12:00

EECS 598-017 Convex Optimization Methods in Control [Seiler] - TTh
12:00-1:30

ME 461 Automatic Control [Barton] - TTh 9:00-10:30

ME 542 Vehicle Dynamics and Control [Orosz] - TTh 2:00-3:30

ME 561 (EECS 561) Design of Digital Control systems [Vasudevan] - TTh
3:00-4:30

ME 564 560 (AERO 550) (EECS 560) (CEE 571) Linear System Theory
[Scruggs] - MWF 9:30-10:30

ME 565 Battery Systems and Control [Siegel] – MW 10:30-12:00

ME 566 Hybrid Electric Vehicles [Grewe, Liu] – Th 6:00-9:00

ME 599 (CEE 501) (ROB 599) (ISD 599) Dynamics and Control of
Connected Vehicles [Orosz] – TTh 12:00-1:30

ROB 550 Robotics Systems Laboratory [Gaskell] – TTh 9:00-10:00

ROB 598 Robot Modeling and Control [Gregg] – TTh 2:30-4:00

Interesting IOE courses

IOE 510 - Linear Programming I

IOE 511- Continuous Optimization Methods

IOE 614 - Integer Programming

AE 740: “MULTI-AGENT CONTROL”

Winter Semester 2020

Instructor:	Dimitra Panagou	Time:	10:30am-12:00pm
Email:	dpanagou@umich.edu	Place:	1012 FXB

Office Hours: TBD.

Objectives: This course is primarily designed for graduate students in control, interested in the fundamentals of multi-agent control, and in recent developments related to safety and resilience for multi-agent systems. The course will focus on providing the fundamental representations and graph-theoretic notions for multi-agent and networked control systems, as well as methods for the control synthesis under safety constraints and uncertain (malicious or faulty) information, with applications in distributed multi-robot systems. Time permitting, the course will overview more recent developments on spatiotemporal control synthesis under safety (state) and temporal (time) constraints, and illustrate their application in multi-agent problems.

Prerequisites: An graduate-level understanding of linear systems and control, state-space methods, and real analysis is assumed, along with working knowledge of MATLAB (or other similar tool/programming language). Please consider the following (strong) recommendations:

- **(Pre-requisite)** It is expected that you have already completed AE 550/EECS 560 “*Linear Systems*” or equivalent.
 - Taking AE 550 in the same semester as AE 740 is **not** sufficient and **not** recommended. If you decide to enroll in AE 740 without having completed AE 550, you are at your own risk.
- **(Co-requisite)** It is expected that you are taking AE 551/EECS 562 “*Nonlinear Systems and Control*” in the same semester as AE 740, or you have already completed AE 551/EECS 562 (or equivalent).
 - Taking AE 740 without taking or having taken AE 551 is **not** recommended. If you decide to take AE 740 without AE 551, you are expected to learn the main topics of Lyapunov theory on your own.
 - MATH 451 (Advanced Calculus I) is a recommended prerequisite for AE 551/EECS 562, and similarly, it is recommended for AE 740.
- **(Recommended)** It is expected that you have already completed at least one additional graduate course in control, e.g., AE 584, AE 580, AE 575, ROB 501, ROB 535, EECS 563, AE 740, ... Note that the provided list is indicative and not thorough.
 - Information about relevant courses is available at <https://controls.engin.umich.edu/control-courses/>

Grading Policy: Homework (90%) (Three Projects (90% = 3 * 30%)), Paper Review and Presentation (10%). The format of the presentation is TBD.

Textbook: None required. Notes will be posted on Canvas after the lectures. Some good references on the topics of the course (not thorough list):

- *Graph Theoretic Methods in Multi-Agent Networks*, by Mehran Mesbahi and Magnus Egerstedt, Princeton Series in Applied Mathematics (Available online from the Michigan Library)
- *Nonlinear Systems, 3rd Edition*, by Hassan K. Khalil, Prentice Hall, 2002
- *Switching in Systems and Control*, by Daniel Liberzon, Birkhäuser, 2003
- *Convex Optimization*, by Stephen Boyd and Lieven Vandenberghe, Cambridge University Press (Available online at <https://web.stanford.edu/~boyd/cvxbook/>)

(Very) Tentative Course Outline and Topics (Time Permitting):

- Introduction to Graph Theory and Networked Control
 - Graph-Theoretical Representations of Networked Systems
 - Consensus / Agreement Protocols
 - Fundamental Properties: Connectivity
 - Applications: Formation Control
- Review of Lyapunov Stability Theory and Switched Systems Theory
 - Review of Lyapunov Functions and Control Lyapunov Functions
 - Control Barrier Functions (Lyapunov-like, Zeroing, Reciprocal)
 - Review of Convex Optimization
 - Synthesis of Safety-Critical Controllers
 - Finite Time Stability (FTS) and Fixed Time Stability (FxTS)
 - Synthesis of Safety-Critical and Time-Critical Controllers
- Safety and Resilience in Multi-Agent Networks
 - Fundamental Properties for Resilience: r -robustness, strong r -robustness
 - Networked Control and Estimation for Resilience
 - Synthesis of Safe and Resilient Networks

Useful Papers: This is a non-inclusive list of relevant papers that will be touched during the course.

- D. Panagou, D. M. Stipanovic and P. G. Voulgaris, *Distributed coordination and control for multi-robot networks using Lyapunov-like Barrier Functions*, IEEE Transactions on Automatic Control, vol. 61, no. 3, pp. 617-632, March 2016
- D. Panagou, *A distributed feedback motion planning protocol for multiple unicycle agents of different classes*, IEEE Transactions on Automatic Control, vol. 62, no. 3, pp. 1178-1193, March 2017
- K. Garg, E. Arabi, D. Panagou, *Prescribed-time control under spatiotemporal and input constraints: A QP-based approach*, <https://arxiv.org/abs/1906.10091>

- S. Prajna, A. Jadbabaie, *Safety Verification of Hybrid Systems Using Barrier Certificates*, In: Alur R., Pappas G.J. (eds) Hybrid Systems: Computation and Control. HSCC 2004. Lecture Notes in Computer Science, vol 2993. Springer, Berlin, Heidelberg
- P. Wieland, F. Allgower. *Constructive safety using Control Barrier Functions*, IFAC Proceedings Volumes, Volume 40, Issue 12, 2007, Pages 462-467
- A. D. Ames, S. Coogan, M. Egerstedt, G. Notomista, K. Sreenath, P. Tabuada, *Control Barrier Functions: Theory and Applications*, <https://arxiv.org/abs/1903.11199>
- A. D. Ames, X. Xu, J. W. Grizzle, P. Tabuada, *Control Barrier Function Based Quadratic Programs with Application to Automotive Safety Systems*, IEEE Transactions on Automatic Control, vol. 62, no. 8, pp. 3861-3876, August 2017
- S. Bhat and D. Bernstein, *Finite-time Stability of Continuous Autonomous Systems*, SIAM Journal of Control and Optimization, Vol. 38, No. 3, pp. 751-766
- A. Polyakov, *Nonlinear feedback design for fixed-time stabilization of linear control systems*, IEEE Transactions on Automatic Control, vol. 57, no. 8, pp. 2106-2010, August 2012.

Academic Honesty: As a University of Michigan engineering student, you have agreed to abide with the CoE's academic honesty policy and the Student Honor Code (see Canvas for more information). Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation.

ROB 599: Robot Modeling and Control

Course Information

Winter 2020

Class Meeting: Tuesday/Thursday 2:30-4pm

Starts: January 8, 2020

Ends: April 21, 2020

Location: 1500 EECS

Instructor: Robert D. Gregg, Ph.D.

Office: 3213 EECS

Email: rdgregg@umich.edu

Office Hours: TBD

Course Pre-requisites, Co-requisites, and/or Other Restrictions

An introduction to Linear Systems Theory at the level of EECS 560 / AERO 550 / ME 564 is necessary. Alternatively, an undergraduate controls class and ROB 501 will suffice. Knowledge of linear algebra and differential equations is required; Knowledge of Matlab and Mathematica is recommended.

Course Description

ROB 599 - Robot Modeling and Control (3 semester hours) Kinematics and dynamics of robots; methods of control; force control; robust and adaptive control; feedback linearization; Lyapunov design methods; passivity and network control; control of multiple and redundant robots; teleoperation.

Student Learning Objectives/Outcomes

Upon successful completion of this course, students will:

- 1. have a basic understanding of the kinematics of robot manipulators and mobile robots.*
- 2. understand the dynamics of Lagrangian mechanical systems and be able to compute the dynamic equations of motion of any robot manipulator.*
- 3. have a basic understanding of nonlinear control methods such as feedback linearization, passivity-based, robust, and adaptive control, and be able to analyze the stability and tracking performance of closed loop systems using Lyapunov theory.*
- 4. be able to model, design, and simulate nonlinear controllers for manipulators and mobile robots.*

Required Textbooks and Materials

(MLS) *R. Murray, Z. Li, and S. Sastry, A Mathematical Introduction to Robotic Manipulation, CRC Press, Boca Raton, FL, 1994.*

Free download: <http://www.cds.caltech.edu/~murray/books/MLS/pdf/mls94-complete.pdf>

(SHV) *Mark W. Spong, Seth Hutchinson, and M. Vidyasagar, Robot Modeling and Control, John Wiley & Sons, Inc., New York, NY, 2006.*

Required Software

MathWorks MATLAB

Access/installation: <https://caenfaq.engin.umich.edu/software-for-students/matlab-for-students>

Wolfram Mathematica

Access/installation: https://kb.lsa.umich.edu/lsait/index.php/Mathematica_Off-Campus_Licenses

Tutorials: <http://www.wolfram.com/broadcast/screencasts/handsonstart/>

Topical Outline

1. *Screw Theory for Kinematics (MLS)*
 - *Rotation Matrices and Homogeneous Transformations*
 - *Exponential Coordinates for Rigid Motion*
 - *Twists and Wrenches*
 - *Forward and Inverse Kinematics*
 - *The Manipulator Jacobian*
2. *Dynamics (MLS/SHV)*
 - *The Euler-Lagrange Equations*
 - *The Dynamics of Example Manipulators*
 - *Properties of Manipulator Dynamic Equations*
3. *Independent Joint Control (SHV)*
 - *Actuator Dynamics*
 - *Set-Point Control*
 - *Feedforward Control*
 - *Drive-Train Dynamics*
 - *State Space Design*
4. *Multivariable Nonlinear Control (SHV)*
 - *Lyapunov Stability and the Invariance Principle*
 - *PD Control*
 - *Inverse Dynamics*
 - *Passivity-Based Robust and Adaptive Control*
 - *Force Control*
 - *Energy Shaping*
5. *Geometric Nonlinear Control (SHV)*
 - *Vector Fields and Distributions*
 - *The Frobenius Theorem*
 - *Feedback Linearization*

6. *Introduction to Mobile Robots (SHV)**

- *Nonholonomic Constraints*
- *Kinematics and Dynamics of Mobile Robots*
- *Motion Planning and Obstacle Avoidance*
- *Controllability and Chow's Theorem*
- *Controller Design for Mobile Robots*

* if time permits

Exams and Grading Policy

There will be homework assignments involving worked problems and computer simulations, a midterm exam, and a final exam. Grades will be determined based on the following formula:

Homework 20%

Midterm Exam 40%

Final Exam 40%

HW Scoring: (each problem)

- a) **three** (3) points if the problem is perfectly correct or nearly so. Of course, "nearly so" is a subjective evaluation. I don't consider a numerical mistake to be important if it doesn't change the basic problem nor lead to greatly simplified reasoning. I am always concerned about conceptual errors.
- b) **two** (2) points if there are several minor errors or at least one major error, but it is clear that the person had a good idea of how to work the problem
- c) **one** (1) point if the problem was attempted, but the reasoning is quite wrong, quite incomplete, or if the solution was unreadable (illegible writing, undefined notation, etc.)
- d) **zero** (0) points only if the problem was not attempted.

The total number of points for each HW set will vary because the number of assigned problems will vary. When computing HW averages, each HW set will be normalized to a score of 100%. If you have a concern with your HW grading, please talk to the GSI and/or grader first. Professor Gregg will only get involved after you give them a chance to address the concern.

OTHER COURSE POLICIES

Make-up Exams: No make-up exams will be given. In the event of an excused absence (illness with doctor's note, job-related travel, holy day absence, etc.; Proper documents should be provided), the weight of the exam will be shifted to the remaining exam.

Late Work: Not acceptable

Class Attendance: It is your responsibility to take notes. Lecture notes will not be provided. In the event of an absence, please watch the recorded lecture.

Classroom Citizenship: Professional at all times. Please do not have a conversation your neighbor during lecture. Smelly foods are distracting to others and will not be allowed in the classroom. As courtesy to classmates and instructor, electronic devices should be turned off during class except when used for note taking or in-class exercises. Cell phone use (e.g., texting) is disruptive to class and will not be tolerated.

Disability Statement: The University of Michigan is committed to providing equal opportunity for participation in all programs, services and activities. Request for accommodations by persons with disabilities may be made by contacting the Services for Students with Disabilities (SSD) Office located at G664 Haven Hall. The SSD phone number is 734-763-3000. Once your eligibility for an accommodation has been determined you will be issued a verified individual services accommodation (VISA) form. Please present this form to me at the beginning of the term, or at least two weeks prior to the need for the accommodation (test, project, etc...).

Inclusion Statement: It is my intention that students from all backgrounds and perspectives will be well served by this course, and that the diversity that students bring to this class will be viewed as an asset. I welcome individuals of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, socioeconomic background, family education level, ability – and other visible and nonvisible differences. All members of this class are expected to contribute to a respectful, welcoming, and inclusive environment for every other member of the class. Your suggestions are encouraged and appreciated.

These descriptions and timelines are subject to change at the discretion of the Professor.

Instructors: Prof Gábor Orosz
Dept. of Mechanical Engineering
Autolab G034
orosz@umich.edu

Mr Hao Wang
Dept. of Mechanical Engineering
Autolab G041
haowangm@umich.edu

Lectures: Tu 12:00pm - 1:30pm, CHRYS 151
Th 12:00pm - 1:30pm, CHRYS 151

Recitation: Fr 3:30pm - 5:00pm, CHRYS 151

Distance learning students are required to attend the office hours in **at least every second week**.

Office hours: M 10:00am - 11:00am
Tu 5:00pm - 6:00pm,
We 9:00am - 10:00am, Autolab G034
We 4:00pm - 6:00pm
We 7:00 pm - 8:00pm, Bluejeans
Th 8:00 pm - 9:00pm, Bluejeans

Prerequisites: You are expected to have knowledge of differential equations, linear algebra, and Laplace or Fourier transform.

Reading: Materials will be provided as the course progresses

Course description: This course focuses on modeling and control of connected vehicle systems consisting of human driven and connected automated vehicles. Models are built in terms of ordinary differential equations and delay differential equations. The stability of uniform flow equilibrium studied at the linear and nonlinear levels. Controllers for connected automated vehicles are designed so that they can ensure stability and disturbance attenuation around the equilibrium. The impacts of utilizing connectivity in order to ensure traffic safety and efficiency are highlighted.

The Engineering Honor Code: <https://elc.engin.umich.edu/honor-council/>

No member of the community shall take unfair advantage of any other member of the community.

Homework Assignments: Nine homework assignments will be set during the term that will be posted on the course's website. Homework sets are **due no later than the start of class on Thursdays in paper format**. For distance learning students the deadlines are extended until **Sunday midnight EST** and they shall upload the scanned homeworks named **HW##_firstname_lastname.pdf** on canvas. Late homeworks are accepted up to 72 hours after the deadline but 50% of the grade will be taken off. The lowest homework score for the term will be dropped. Homework solutions will be available through the course web site.

You are encouraged to discuss and work on homework together but the final document must represent your own understanding of the material.

If you find errors in your graded homework (e.g. scores do not add up, the grader missed a page etc.) you may ask for re-grade. You need to attach a sheet where you write up the issue and resubmit the homework to the professor within one week after receiving the graded homework.

Examinations: Midterm Exam 1: Feb 17-19 (Mon-Wed)
Midterm Exam 2: Mar 30 - Apr 1 (Mon-Wed)

The exams will be closed book. One sheet of notes (8.5" by 11") will be permitted for the exams (one-sided for the midterm and double-sided for the final).

Grading:	Homework	25%
	Midterm Exam 1	25%
	Midterm Exam 2	25%
	Project	25%

Additional rules: no laptops, cell phones, iPods, iPads, etc. during the class

LECTURE	DATE	TOPICS	READING	HW DUE DATES
1	Th 1/9	Car-following and continuum traffic models		
2	Tu 1/14	Plant stability and string stability		
3	Th 1/16	Ring configuration		HW#01
4	Tu 1/21	Time delay systems		
5	Th 1/23	Time delay systems		HW#02
6	Tu 1/28	Car-following with reaction time		
7	Th 1/30	Car-following with reaction time (first order lag approximation)		HW#03
8	Tu 2/4	Adaptive cruise control – including physics		
9	Th 2/6	Adaptive cruise control – including physics		HW#04
10	Tu 2/11	From adaptive to connected cruise control – digital effects		
11	Th 2/13	From adaptive to connected cruise control – digital effects		HW#05
12	Tu 2/18	Connected cruise control – acceleration feedback		
	M-W 2/17-2/19	MIDTERM EXAM 1		
13	Th 2/20	Experimental Session		
14	Tu 2/25	Head-to-tail string stability (3-car example)		
15	Th 2/27	Head-to-tail string stability (network control)		HW#06
	3/2–3/6	WINTER RECESS		
16	Tu 3/10	Robust control of connected vehicle systems		
17	Th 3/12	Connectivity-based energy efficiency		HW#07
18	Tu 3/17	Numerical continuation		
19	Th 3/19	Nonlinear dynamics of connected vehicles (ring vs open chain)		HW#08
20	Tu 3/24	Chaozhe He Connected cruise control for heavy duty vehicles		
21	Th 3/26	Ilya Kolmanovsky Energy/fuel efficiency considerations in connected and autonomous driving		HW#09
22	Tu 3/31	Tulga Ersal Connected testbeds for connected vehicles		
	M-W 3/30-4/1	MIDTERM EXAM 2		

23	Th 4/2	Henry Liu TBA	Project Proposal
24	Tu 4/7	Ram Vasudevan Bridging the Gap Between Safety and Real-Time Performance for Autonomous Systems	
25	Th 4/9	Yiheng Feng Smart transportation infrastructure: modeling and applications	Project Milestone 1
26	Tu 4/14	Yafeng Yin Cooperative truck platooning for energy savings: path planning, formation and benefit redistribution	
27	Th 4/16	Neda Masoud An optimization framework for peer-to-peer ridesharing	Project Milestone 2
28	Tu 4/21	Review and Project Presentation	

HW#01 – Dynamics and control

HW#02 – Plant stability and string stability

HW#03 – Time delay systems

HW#04 – Human car following

HW#05 – Adaptive cruise control

HW#06 – Connected cruise control – digital effects and acceleration feedback

HW#07 – Connected cruise control – networked design and robustness

HW#08 – Connected cruise control – safety and energy efficiency

HW#09 – Nonlinear effects

EECS 598: Convex Optimization Methods in Control

Instructor: Pete Seiler

Credits: 3

Pre-requisite: EECS 560 (AERO 550) (ME 564) Linear Systems Theory or instructor permission.

Meets: Tu/Th 12:00PM - 1:30PM in 1690 BEYSTER

Course Description

Convex optimization plays a central role in the numerical solution of many design and analysis problems in control theory. This course focuses on the practical aspects of using convex optimization methods to solve these problems. First, the basic properties and theory of convex optimization will be introduced. This will include techniques to formulate engineering problems as convex optimizations. Existing software and implementation issues will also be discussed. Next, we will explore the ties between convex optimization and problems in control theory. This will include methods to assess robustness of systems with nonlinearities, parametric uncertainty and unmodeled dynamics. It will also include methods to solve optimal control problems, e.g. H-infinity design. The course will conclude with more recent results in nonlinear analysis, e.g. sum-of-squares programming. Example problems of industrial relevance will be used to highlight the utility of these methods.

Course Topics

Introduction

1: Course Overview

Essentials of Convex Optimization

2: Convex Sets

3: Convex Functions

4: Convex Optimization Problems

5: Numerical Tools

6: Duality

7: Unconstrained Minimization

8: Constrained Minimization

Nominal Analysis & Synthesis

9: Internal Stability

10: State Feedback Synthesis

11: Bounded Real Lemma

12: H-infinity State Feedback Synthesis

13: H-infinity Optimal Control

14: H2 Optimal Control

15: Linear Parameter Varying Systems

Robust Analysis and Synthesis

- 16: Modeling Uncertain Systems
- 17: Circle Criterion
- 18: Analysis with Unmodeled (Dynamic) Uncertainty
- 19: Analysis with Dynamic LTI Uncertainty
- 20: Pointwise Frequency Domain Constraints
- 21: Analysis with General Integral Quadratic Constraints
- 22: Application to analyzing convergence rates for first-order optimization methods

Nonlinear Analysis with Sum-of-Squares

- 22: Multivariable Polynomials
- 23: Multivariable Polynomials
- 24: Sum-of-Squares Optimization
- 25: Sum-of-Squares Toolbox
- 26: Nonlinear Analysis with SOS
- 27: Nonlinear Analysis with SOS

Miscellaneous Topics (Time Permitting)

- Youla Parameterization and Optimization with Ritz Approximation
- Model Reduction
- Nonconvex QCQPs

References

1. *Convex Optimization* by Boyd and Vandenberghe
(<http://www.stanford.edu/~boyd/cvxbook/>)
2. *Linear Matrix Inequalities in System and Control Theory* by Boyd, El Ghaoui, Feron, and Balakrishnan (<http://www.stanford.edu/~boyd/lmibook/>)
3. "System analysis via integral quadratic constraints," Megretski and Rantzer, IEEE Transactions on Automatic Control, Vol. 42, # 6, June 1997, pp. 819-830.
4. *Structured Semidefinite Programs and Semialgebraic Geometry Methods in Robustness and Optimization* by Parrilo (Ph.D. Thesis, California Institute of Technology May 2000). (<http://www.mit.edu/~parrilo/pubs/index.html>)
5. *Linear Controller Design – Limits of Performance* by Boyd and Barratt
(<http://www.stanford.edu/~boyd/lcdbook/>)

EECS 598: Reinforcement Learning Theory (Tentative)

Instructor: Lei Ying (leiying@umich.edu)

Credits:3

Prerequisite: EECS 502 (**required**)

Meets: TTH 10:30-12PM; 1005 DOW

Topics: This course covers fundamental theories and principles of reinforcement learning. Topics to be covered include:

1. Dynamic programming and the principle of optimality
2. Multi-armed bandit: epsilon-greedy, Upper Confidence Bound (UCB) algorithm, Thompson Sampling
3. Markov chains and Markov Decision Process (MDP)
4. Value iteration, policy iteration, and LP formulation
5. Q-Learning: Model-based and model-free
6. Linear function approximation and deep reinforcement learning
7. Temporal-difference learning
8. SARSA
9. Policy gradient algorithm and variance reduction
10. The ODE methods and convergence analysis